LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2024**

UMT 1501 – ALGEBRA

Date: 09-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)	
	Answer ALL the Questions -	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	Identify the cubic equation which shall have 1, $3-\sqrt{-2}$ as its roots	
b)	Write the uses of Horner method.	
c)	Use Binomial Theorem to find the 3rd power of 11.	
d)	Write down the Cayley Hamilton theorem.	
e)	Find the number of divisors of 360.	
2.	**Fill in the blanks**	
a)	If $f(x)$ is a polynomial, then $f(a)$ is the _____ when $f(x)$ is divided by $x-a$.	
b)	The main principle used in Horner's method is _____.	
c)	The expansion of $(x+a)^n$ is _____.	
d)	The condition for two matrices to be similar is _____.	
e)	Every composite number can be resolved into prime factors and this can be done in ____ way/ways.	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**MCQ**	
a)	Which of the following roots occurs in pairs for an equation with real coefficients? i) Real and imaginary ii) Real and irrational iii) Imaginary and irrational iv) None of these	
b)	If a,b,c are the roots of a reciprocal equation, then _____ are also roots. i¿ $-a,-b,-c$ ii¿ $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ iii) Both i and ii iv) None of these.	
c)	$\frac{e^x+e^{-x}}{2}=$¿ i) $1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\ldots\infty$ ii) $1-\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\ldots\infty$ iii) $1+\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\ldots\ldots\infty$ iv) $x+\frac{x^3}{3!}+\frac{x^5}{5!}\ldots\ldots\ldots\infty$	
d)	The sum of the eigen value of the matrix $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is	

	i) 6	ii) 4	iii) 8	iv) -8
e)	Which of the following is the smallest number with 18 divisors?			
	i)180	ii)268	iii)120	iv)162

4.	**True or False**
a)	Any value x for which the polynomial $f(x)$ vanishes is called a root of the equation.
b)	An equation $f(x)=0$ can have more positive roots than the total number of sign changes in $f(x)$.
c)	The number of terms in the binomial expansion of $(x+a)^n$ is n.
d)	If A and B are similar matrices then they have the same characteristic equation.
e)	n^2+n+41 is a prime number if n<52.

SECTION B - K3 (CO2)

Answer any TWO of the following **(2 x 10 = 20)**

5.	Solve the equation $x^4-5x^3+4x^2+8x-8=0$ of which one root is $1-\sqrt{5}$.
6.	Solve the $81x^3-18x^2-36x+8=0$ whose roots are in harmonic progression.
7.	Sum the series $\frac{15}{16}-\frac{15.21}{16.24}+\frac{15.21.27}{16.24.32}-\cdots$ to ∞.
8.	Determine the characteristic equation of the matrix $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and hence find A^5.

SECTION C – K4 (CO3)

Answer any TWO of the following **(2 x 10 = 20)**

9.	The conditions that the roots of the equation $ax^4+4bx^3+6cx^2+4dx+e=0$ having two pairs of equal roots are $3abc=a^2d+2b^3$ and $eb^2=ad^2$. Justify your answer.
10.	Estimate the sum of the eleventh powers of the roots of the equation $x^7+5x^4+1=0$.
11.	Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$.
12.	Show that $13^{2n+1}+9^{2n+1}$ is divisible by 22.

SECTION D – K5 (CO4)

Answer any ONE of the following **(1 x 20 = 20)**

13.	a.) Predict all the roots of the equation $6x^5-x^4-43x^3+43x^2+x-6=0$. **(12 marks)** b.) Test whether the relation $\log\sqrt{12}=1+\left(\frac{1}{2}+\frac{1}{3}\right).\frac{1}{4}+\left(\frac{1}{4}+\frac{1}{5}\right).\frac{1}{4^2}+\left(\frac{1}{6}+\frac{1}{7}\right).\frac{1}{4^3}+\cdots$ is valid **(8 marks**
14.	Diagonalize the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$

SECTION E – K6 (CO5)

Answer any ONE of the following **(1 x 20 = 20)**

15.	Using Horner's method, predict the positive root of $x^3-2x^2-3x-4=0$ which lies between 3 and 4, correct to three decimal places
16.	a.) Sum the series $\sum_{n=0}^{\infty}\frac{n^2+3}{n+2}.\frac{x^n}{n!}$. **(10 Marks)** b.) State and prove Wilson's theorem. **(10 Marks)**

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